Calculus Summer Assignment

This review packet covers material that will be given on a test to AP Calculus students during the first week of class. A topic list is below, followed by a written assignment. You should expect that there will be little or no in-class review for this test. Look the list over several times during the summer. Work out the problems sometime in August to make sure that you have a command of everything. If you get stuck on something, you will find an almost unlimited number of places online where you can get help. Feel free to use them. The internet is a great resource. There are videos on every aspect of math you are likely to encounter during your lifetime.

Students who take the assignment seriously will do well on the September test. And for the past 18 years at GSA, no student who has neglected this assignment has successfully completed AP Calculus.

As you know, our class will be meeting 3 times per week for 65 minutes (BC will meet for an additional 3 classes per week, but only for the second semester.) This is an unusually small amount of time for an AP Calculus class. As such, we need to hit the ground running and don't have the luxury of taking two or three weeks to review aspects of pre-calculus that you have worked through in the past. In particular, trigonometry and logarithms appear throughout calculus, in both expected and unexpected ways. Therefore, it is very important that you show up for class in September with these topics pretty well in hand.

A note on **Graphing calculators**

A graphing calculator is required for AP Calculus. It is used extensively, and you are not allowed to sit for the AP exam in May without one.

We recommend the **TI-nspire**. We will use it frequently, beginning fairly early in the school year, and its use will be demonstrated in class. We have some available if you would like to borrow one for the school year. If you do borrow one, you will be responsible for returning it at the end of the year. If you wish to buy your own you may do so. In that case, please contact Mr. Bender so he can point you toward the right calculator at the best price.

Other graphing calculators may be fine (and some are cheaper), but don't expect your teacher to know how to use every calculator made. **Important:** If you are planning to purchase/use a different calculator, download the AP Calculus course description from the College Board website and make sure that the calculator is on the list of models approved for the course. In particular, you can't use a calculator app on a cell phone or tablet, and you can't use a calculator with a QWERTY keyboard. Most major calculators are fine, but don't make a mistake with such an expensive purchase.

You should be absolutely solid on the following:

1) **Trigonometry**:

a) The definitions of the six basic trig functions, both in terms of triangles (SOHCAHTOA) and of the unit circle.

b) Solving right triangles using the definitions of the trig functions.

c) Solving triangles using the ***Law of Sines*** and the ***Law of Cosines.***

d) Definition of radian measure; conversion between degrees and radians. You should also know the radian value of commonly used angles: 0, 30, 45, 60, 90 and related larger angles. These should be <u>automatic</u> to you. (Please note: In calculus, we measure almost all angles in radians.)

e) Basic trig identities, including the following:

Pythagorean identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \tan^2 x = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$

Angle addition formulas

$\sin(x+y) = \sin x \cos y + \sin y \cos x$ $\sin(x-y) = \sin x \cos y - \sin y \cos x$ $\cos(x+y) = \cos x \cos y - \sin x \sin y$ $\cos(x-y) = \cos x \cos y + \sin x \sin y$	More compactly: $\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$ $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

Double-angle formulas

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$ (Know all 3 forms)

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

Half-angle formulas

$\sin \dfrac{x}{2} = \pm\sqrt{\dfrac{1-\cos 2x}{2}}$ $\qquad \cos \dfrac{x}{2} = \pm\sqrt{\dfrac{1+\cos 2x}{2}}$

$\tan \dfrac{x}{2} = \pm\sqrt{\dfrac{1-\cos x}{1+\cos x}} = \dfrac{\sin x}{1+\cos x} = \dfrac{1-\cos x}{\sin x}$

f) Finding the sine, cosine, and tangent of particular angles (0 radians, $\pi/4$ radians, $2\pi/3$ radians, etc.). **These should be <u>automatic</u> to you.**

g) Given a circle of radius *r* and a sector subtended by an angle of θ radians, what is the area of the sector? (We will need this formula during the course, and it will be on your summer-work test.)

h) If you know the value of one of the trig functions for a right triangle, you can find the others easily. E.g.: Suppose that you know that $\sin A = \frac{x}{5}$. Draw a right triangle and label one of the acute angles as *A*. Label the leg opposite *A* as *x* and label the hypotenuse 5. This gives us $\sin A = \frac{x}{5}$. Find the length of the third side of the triangle by using the Pythagorean theorem. Now it should be easy to use the SOHCAHTOA relationships to find the values of the other five trig functions of *A*.

2) **Lines** – you should be absolutely solid on the following:

a) Finding the point-slope and slope intercept forms of the equation of a line given either the slope and a point, or two points

b) Find the equation of a perpendicular to a line through a given point

3) **Graphs** – you should be familiar with the graphs of the following functions and their various transformations:

a) $y = a|x-h|+k$

b) $y = a(x-h)^2 + k$

c) $y = x^3$

d) $y = \sqrt{x}$; $y = \sqrt[3]{x}$

e) $y = a^x$

f) $y = \log_a x$

g) $y = \sin x$; $y = \cos x$; $y = \tan x$

4) **Evaluating functions**. E.g., If $f(x) = x^2$, find $f(7)$, $f(2y)$ and $\frac{f(x+h)-f(x)}{h}$.

5) **Logarithms**: Review the definition of logarithms and be comfortable with the laws of logarithms.

Identities:

1. $\log_b b^x = x$
2. $b^{\log_b x} = x$

Three basic Rules:

3. If $\log_b x = m$ and $\log_b y = n$, then $\log_b xy = m+n$
4. If $\log_b x = m$ and $\log_b y = n$, then $\log_b \frac{x}{y} = m-n$
5. $\log_b x^n = n \log_b x$

Change of Base Rule

6. $\log_b a = \dfrac{\log a}{\log b} = \dfrac{\ln a}{\ln b} = \dfrac{\log_c a}{\log_c b}$. You can change to any base but base 10 and base e are easiest because they are built in functions in most calculators.

Problems:

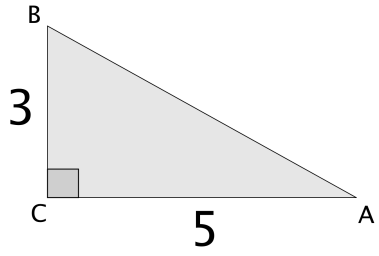
You should be able to do all of the following without a calculator.

1) Suppose $f(x) = x^2 + 5x - 7$.

a) Find $\dfrac{f(x+h) - f(x)}{h}$. Simplify completely.

b) $f(f(4))$.

2) For the triangle at right, find $\sin A$, $\cos A$, $\tan A$, $\csc A$, $\sec A$, and and $\cot A$. Leave your answers as fractions properly simplified.



3) Line L passes thru the points (2, 6) and (4, 12).

a) Find the equation of L in point-slope form.
b) Find the equation in point-slope form of the line perpendicular to L and passing through the point (2, 11).

4) Sketch the graph of $y = 4|x+5| - 7$. Mark the vertex with its coordinates.

5) Sketch the graph of $y = -2(x-3)^2 + 8$. Mark the vertex with its coordinates.

6) Convert 72 degrees to radians.

7) Convert $\dfrac{5\pi}{4}$ radians to degrees.

8) Suppose that A, B, and C are measures of angles in a triangle with sin A = 0.3, cos B = 0.8, and tan C = 2.8. Use appropriate trig identities to find the <u>exact</u> values of

a) cos A

b) sin B

c) sin (A – B)

d) cos (A + B)

e) csc A

f) sec C

g) sin 2B + cos 2A

h) $\sin^2 (A/2)$

i) $\cos^2 (A/2)$

9) State the three Pythagorean identities.

10) Fill in the table, All angles are measured in radians.

Angle	**sine**	**cosine**	**tangent**
0			
$\frac{\pi}{6}$			
$\frac{\pi}{3}$			
$\frac{\pi}{2}$			
$\frac{2\pi}{3}$			
π			
$\frac{7\pi}{4}$			
$\frac{3\pi}{2}$			
2π			

11) Write as a single logarithm. Simplify as much as possible.

a) $\log 25 + \log 4$

b) $\log 60 - \log 6$

c) $2 \ln 6 + \ln 11$

12) Simplify completely.

a) $\log 1000$

b) $4^{3/2}$

c) $e^{\ln 63}$

d) $\log_5 25$

e) $\log 0.1$

f) $\log 10^{14}$

g) $\log(-6)$

13) $\log 864$ is between what 2 consecutive integers?

14) A circle has radius 20 cm. A sector of the circle is *subtended* by an angle of $\pi/9$ radians. What is the area of the sector?

15) **Think** about what the graph of the function $f(x) = \dfrac{20}{1+x^2}$ looks like.

- What happens when x gets very big?
- What is the maximum value of $f(x)$ and where does it occur?
- What kind of symmetry does the graph have?

After **think**ing about these things, sketch the graph. Then check your answer with a graphing calculator or a piece of graphing software such as GeoGebra or Desmos.

This function appeared on the free-response section of the 2007 AP Calculus test. At a conservative estimate, 100,000 students lost a lot of credit on the problem because they tried to draw the graph on their calculators instead of **think**ing about the graph first: A minute of **think**ing was all that would have been necessary. But we really don't have to **think** anymore. All we have to do is push the buttons on our calculator and watch the pretty flashing numbers. Pretty flashing numbers… So pretty... **Think**ing bad. Pretty flashing numbers good. Soooooo pretty….

(Note: the problem above was written by Mr. Gilden, but you can probably expect a similar crazy-old-math-teacher sense of humor from Mr. Bender.)